# Comparison of damage localization in mechanical systems based on Stochastic Subspace Identification method

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## Context

- Vibration-based damage localization on structures relates to the monitoring of the changes in the modal parameters
- Based on finite Element model of the structure and output-only measurement data in the reference and damaged states
- Comparison of Stochastic Dynamic Damage Locating Vector (**SDDLV**) approach and Subspace Fitting (**SF**) method, for damage localization

## Covariance computation

- System matrices are subject to uncertainties due to unknown excitation, noise and finite data length
- Estimation from subspace identification, based on covariance-driven Hankel matrix $\mathcal{H}$
- Let $f$ be a function of $\mathcal{H}$, then $\mathrm{cov}(f(\mathcal{H})) \approx \mathcal{J}_{f,\mathcal{H}} \hat{\Sigma}_{\mathcal{H}} \mathcal{J}_{f,\mathcal{H}}^T$ with sensitivity $\mathcal{J}_{f,\mathcal{H}} = \partial f(\mathcal{H}) / \partial \mathrm{vec}(\mathcal{H})$

## SDDLV

$$\textbf{Transfer matrix} \begin{cases} G(s) = R(s) D_c \\ R(s) = C_c (sI - A_c)^{-1} \begin{bmatrix} C_c A_c \\ C_c \end{bmatrix}^{\dagger} \begin{bmatrix} I \\ 0 \end{bmatrix} \end{cases}$$

### Damage localization procedure

- From data: changes in the transfer matrix between both reference and damaged state: $\partial R(s)^T = \hat{R}(s)^T - R(s)^T$ load vector $v(s)$ in the null space of $\partial R(s)^T$
- From FE model: apply load vector $v(s)$ to model
  Stress computation: $S(s) = \mathcal{L}_{\mathrm{model}}(s) v(s)$
- Damaged element indicated by stress $= 0$ (or close to 0)

### Uncertainty propagation and covariance computation

- Computed stress for damage localization is afficted with uncertainties
  - Sensitivity-based uncertainty propagation from measurement data ($\mathcal{H}$) to computed stress

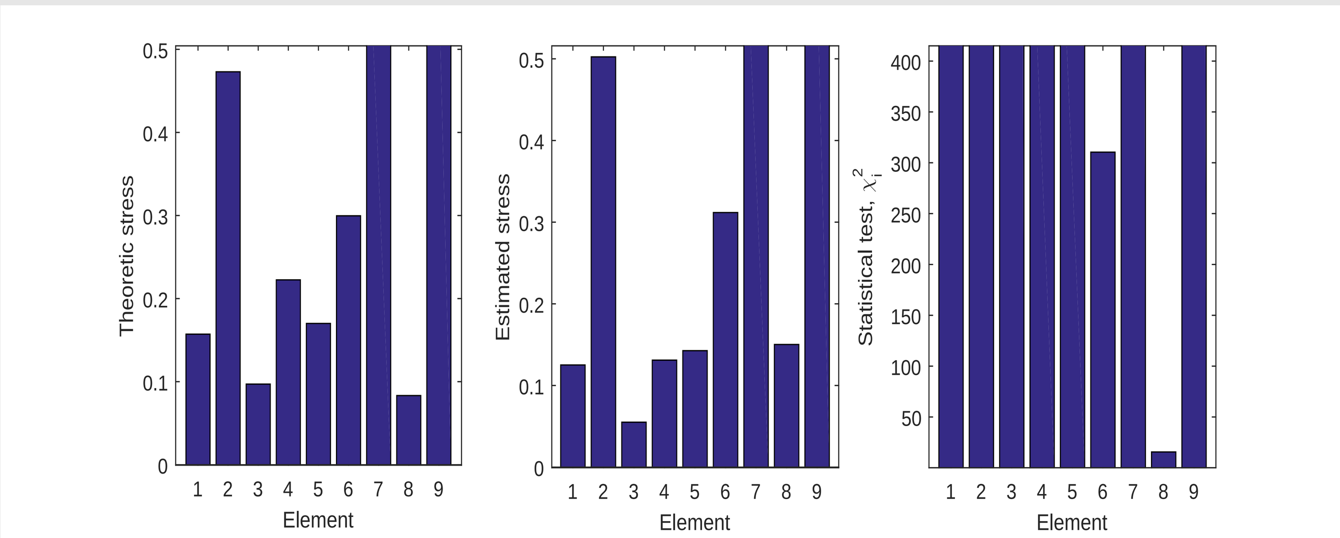
Reference state: $\mathcal{H}^{\mathrm{ref}} \xrightarrow{\mathcal{J}_{A,\mathcal{H}}} A, C \xrightarrow{\mathcal{J}_{R,A}} R$

Damaged state: $\mathcal{H}^{\mathrm{dam}} \xrightarrow{\mathcal{J}_{\tilde{A},\tilde{\mathcal{H}}}} \tilde{A}, \tilde{C} \xrightarrow{\mathcal{J}_{\tilde{R},\tilde{A}}} \tilde{R}$

$$\delta R \xrightarrow{\mathcal{J}_{v,R}} v(s) \xrightarrow{\mathcal{J}_{S(s)}} S(s)$$

$$\mathrm{cov}(S(s)) = \mathcal{J}_{S(s)} (\mathrm{cov}(\mathrm{vec}(\tilde{R}^T)_{\mathrm{re}})) + (\mathrm{cov}(\mathrm{vec}(\tilde{R}^T)_{\mathrm{re}})) \mathcal{J}_{S(s)}^T$$

- $\chi^2$ test $S_i^T \mathrm{cov}(S_i)^{-1} S_i$ for each element $i$




## Mathematical model

**Mechanical model**

$$M\ddot{v}(t) + C\dot{v}(t) + Kv(t) = u(t)$$

**State space model**

$$\begin{cases} \dot{x}(t) = Ax(t) + B_c e(t) \\ y(t) = C_c x(t) + D_c e(t) \end{cases}$$

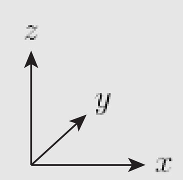
## Subspace Identification

$$\mathcal{Y}^- = \begin{bmatrix} \mathbf{y}_q & \mathbf{y}_{q+1} & \cdots & \mathbf{y}_{N+q-1} \\ \mathbf{y}_{q-1} & \mathbf{y}_q & \cdots & \mathbf{y}_{N+q-2} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{y}_1 & \mathbf{y}_2 & \cdots & \mathbf{y}_N \end{bmatrix}, \quad \mathcal{Y}^+ = \begin{bmatrix} \mathbf{y}_{q+1} & \mathbf{y}_{q+2} & \cdots & \mathbf{y}_{N+q} \\ \mathbf{y}_{q+2} & \mathbf{y}_{q+3} & \cdots & \mathbf{y}_{N+q+1} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{y}_{q+p+1} & \mathbf{y}_{q+p+2} & \cdots & \mathbf{y}_{N+p+q} \end{bmatrix}.$$

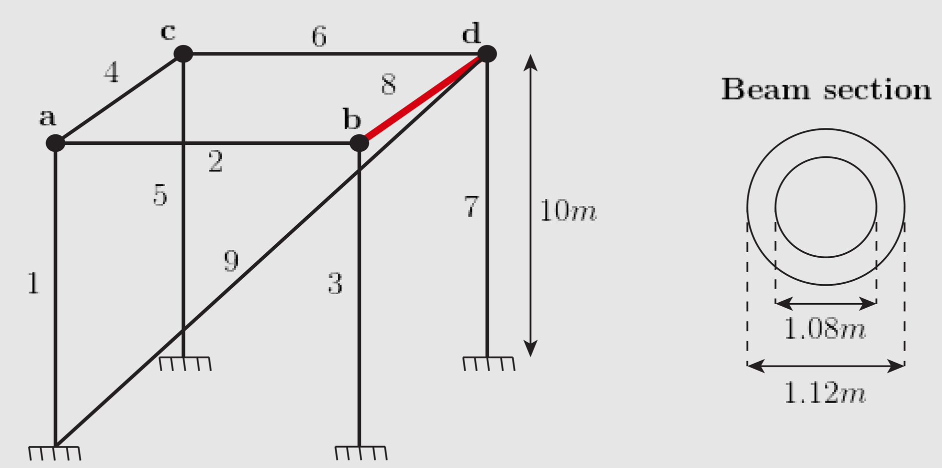
$$\hat{\mathcal{H}} = \frac{1}{N} \mathcal{Y}^+ (\mathcal{Y}^-)^T = \mathbf{U}\Delta\mathbf{V}^T = [\mathbf{U}_1 \ \mathbf{U}_0] \begin{bmatrix} \Delta_1 & \mathbf{0} \\ \mathbf{0} & \Delta_0 \end{bmatrix} \mathbf{V}^T \approx \mathbf{U}_1 \Delta_1 \mathbf{V}_1^T, \quad \hat{\mathcal{O}} = \mathbf{U}_1$$

## Application

- Number of degree of freedom: 24
- Damping ratio = 2%, noise = 5%
- Output sensors: $x$ and $y$ directions at node a and d
- Number of sample: 200000
- Damaged state: decreasing 25% Young modulus at element 8




| | | |
|---|---|---|
| Young's modulus | 210 | GPa |
| Density | 7800 | kg/m3 |
| Poisson ratio | 0.3 | |




## SF

$$\textbf{Observability matrix} \qquad \mathcal{O} = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^p \end{bmatrix}$$

### Damage localization procedure

- FE model updating procedure to identify structure parameters:
  $\boldsymbol{\theta} = \mathrm{argmin}\, ||\mathbf{r}||_2^2$
  $\mathbf{r} = \left[\mathbf{I}_{2n} \otimes (\mathbf{I}_{(p+1)r} - \hat{\mathcal{O}}\hat{\mathcal{O}}^{\dagger})\right] \mathrm{vec} \left\{\mathcal{O}^h(\boldsymbol{\theta}^h)\right\}$
  $\boldsymbol{\theta}_k = \boldsymbol{\theta}_{k-1} - \mathcal{J}_{\mathbf{r}_k}^{\dagger} \mathbf{r}_k$
- $\boldsymbol{\theta}$ related to damage (element stiffness reduction)

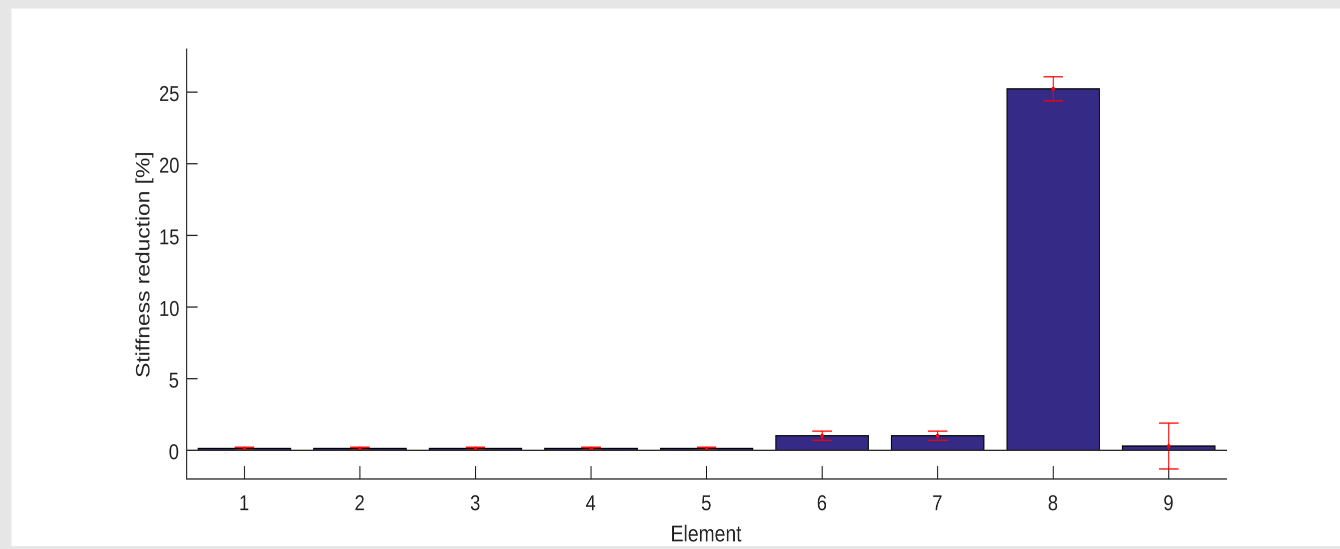
### Uncertainty propagation and covariance computation

- Updating parameters for damage localization is afficted with uncertainties
  - Sensitivity-based uncertainty propagation from measurement data ($\mathcal{H}$) to updated parameters, through each iteration step of the updating minimization problem

Reference state: $\mathcal{H}^{\mathrm{ref}} \xrightarrow{\mathcal{J}_{\boldsymbol{\theta}^{\mathrm{ref}},\mathcal{H}}} \boldsymbol{\theta}^{\mathrm{ref}}$

Damaged state: $\mathcal{H}^{\mathrm{dam}} \xrightarrow{\mathcal{J}_{\boldsymbol{\theta}^{\mathrm{dam}},\mathcal{H}}} \boldsymbol{\theta}^{\mathrm{dam}}$

- Damaged if $\theta_i^{\mathrm{dam}} \pm \sigma_{\theta_i^{\mathrm{dam}}} - \theta_i^{\mathrm{ref}} > \sigma_{\theta_i^{\mathrm{ref}}}$ for each element $i$




## References

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